A WAVE REFRACTION ANALYSIS FOR AN AXIALLY SYMMETRICAL ISLAND

RONALD J. FORST

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A Thesis

By

LIEUTENANT RONALD J. FORST

UNITED STATES NAVY

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LIST OF SYMBOLS

A	Wave amplitude.
b	Distance between adjacent rays.
C	Wave phase speed.
E	Mean wave energy per unit surface area.
g	Acceleration due to gravity.
H	Wave height.
h	Mean depth of water.
I	Travel time.
k	Wave number ($2\pi/L$).
L	Wave length.
m	Slope of the ocean bottom.
n	Ratio of group speed to phase speed.
o	A subscript denoting values at the island shore.
r	Radial coordinate.
c	A subscript denoting a critical value.
T	The wave period.
t	Time.
V	Wave group, speed.
π	3.1415.
ϕ	Phase difference.
K_r	Refraction factor
K_s	Shoaling factor.
ζ	Vertical component of the particle displacement.
Δ	Increment operator.

1 A subscript denoting the outer depth contour or a deep water condition.

ρ Density of sea water (assumed constant).

ds Arc length.

θ Angular coordinate measured counterclockwise from a reference which is parallel to the deep water wave rays.

ABSTRACT

This paper deals with the refraction of long gravity waves about a circular island with uniform slope below mean water level. The Fermat principle is used as a basis to derive an analytical solution for the ray paths. A refraction analysis is performed to determine the refraction factor, phase lag, and wave height at the island shore. The relative power distribution of rays in the far field is determined by considering emergent reflected and refracted waves from the island.

CHAPTER I

INTRODUCTION

1. Statement of the Problem

The purpose of this paper is to study the influence on long gravity waves of an island with circular bottom contours and radially constant slope. Using the principles of wave ray theory, a refraction analysis is carried out to determine the modification of initially plane waves as they move into shallow water about the island. The refraction and direct shoaling effects upon incoming long water waves are considered. The reflected and refracted rays emerging from the island are considered in order to study the divergent wave energy in the far field of the island. This refraction study is a contribution in part to a more exacting treatment of the subject by Vastano (1966), involving a numerical diffraction technique.

2. Historical Review

For centuries man has concerned himself with the effects of islands on surface waves. The early civilizations of Micronesians and Polynesians utilized wave patterns to mark fishing grounds and aid in navigation among the islands. Winkler (1901) recorded some of their techniques through interviews with native chiefs. Krieger (1943) further illustrates this fact and states:

> Every chief and sea pilot possessed elaborate charts based upon his own experience and on knowledge handed down or gained from others. These sea charts are made with thin strips of the midrib of the leaflets of the coconut arranged on a frame usually rectangular in shape. The knowledge they record is indicated by the arrangement of the leaf strips relative to one another and by the forms given to them by bending and crossing. Curved strips indicate the altered direction taken by ocean swells when deflected by the presence of an island; their interactions are nodes where these meet and tend to produce a confused sea.

More recent interest on the specific subject of wave refraction was stimulated by the needs brought about by World War II amphibious operations. As a wave approaches shallow water, its velocity begins to decrease appreciably at a depth of about one-half the wave length and the wave height is influenced by the convergence and divergence of energy along the shore. As the wave bends around the island, "wave shadow" areas may be formed in

the island "wave lee," but often times not to the extent that might be expected. This fact was painfully realized during the invasion of Sicily when the methods of Sverdrup and Munk (1947) were used to forecast wave conditions prior to the landing. Despite the use of these acceptable methods, the wave heights in the lee of the island were greatly underestimated thus severely hampering the landing operation.

The graphical means of constructing wave refraction diagrams was first used by O'Brien (1942). Application to the complex ocean bottoms of nature was made by Johnson, O'Brien, and Isaacs (1948) and Arthur, Munk, and Isaacs (1952). The wave refraction pattern for any bottom topography assumes such that the waves change direction and are bent in such a manner that they tend to assume the slope of the depth contours. Aerial photographs (Munk and Arthur, 1952) qualitatively bear this out. It was learned that the effect of bottom features upon waves depends not upon the absolute depth of water, but upon the depth relative to the wave length. Graphical construction methods by hand are both time consuming and tedious. Griswold (1963) has computer programmed the graphical construction of wave rays and Momoi (1964) has expanded on Griswold's program to include the effects of wave refraction.

In some special cases, analytical solutions to the wave refraction problem have been developed to improve accuracy which can be critical in the investigation of the energy distribution especially on the wave lee side of islands. Analytical and experimental studies of refraction in the region of a caustic have been made by Pierson (1951). A method of conformal transformation was applied by Pocinski (1950). The actual cases treated are restricted by the mathematical complications that are introduced by a particular conformal transformation and by how well the transform simulates a beach contour. Arthur (1950) used Fermat's principle as a basis for an analogy between the refraction and the minimal flight path problem. The effects of current and depth distribution are considered. Johnson (1947) also discusses the refraction of ocean surface waves by current for deep water waves.

Munk and Arthur (1952) applied ray theory to calculate the wave intensity along a refracted ray. This procedure has certain advantages over the usual technique of computing intensity from measurements of the distance between rays; namely in the areas of extreme convergence or divergence.

Analytical solutions for the refraction of waves around an island with concentric circular bottom contours have been obtained by Arthur (1946) through the application

of Fermat's principle. Arthur (1951) determined that the important effects in the penetration of wave energy into the "wave lee" of an island are generally the result of refraction by underwater topography and variability in direction.

3. General Description of the Island

The island bottom configuration selected for the present study is axially symmetrical (i.e., the depth contours are concentric circles.) At 5 km. radius from the island axis the topography drops vertically from mean sea level to a depth of 0.5 km., then assumes a slope of 1:10 out to 41 km. radius, beyond which the depth remains 4.1 km. (see Figure 1). The periods of the waves selected are 2,4,8, and 12 minutes to provide a variation in wave lengths within the tsunami class.

There was little attempt here to simulate the detailed underwater topographical features of any particular island. However, a computer program for the study of the interaction of monochromatic waves with an island of irregular shape is being developed by Reid and Vastano (1966). This program permits depth variation and the use of a special orthogonal system.

It is interesting to note that the selected island configuration lends itself to a fairly simple mathematical treatment. However, if a case were chosen where the extended slope line intersects the mean sea level at other than the island vertex, then a somewhat more formidable problem presents itself with regard to wave amplitude determination and is worthy of further investigation.

4. Assumptions

In water of constant depth the wave celerity, C, for sinusoidal gravity waves of small amplitude, neglecting surface tension, is given by the classical relation (Lamb, 1932)

$$C^2 = (g/k) \tanh kh \, , \tag{1}$$

where h is the mean depth of water, $k = 2\pi/L$, the wave number, and g the acceleration due to gravity. A wave schematic is shown in Figure 2, where the wave height H is equal to twice the amplitude, i.e. the vertical distance between two successive crests, and the period T is the time interval between the passage of two consecutive crests. For any wave greater than 0.1 feet in length, the effect of surface tension on wave speed can be ignored (Wiegel, 1964). The wave speed is related to wave length and period by

$$L = CT. \tag{2}$$

There are two limiting cases of (1) which are appropriate to the so called "deep" and "shallow" water waves respectively. For deep water, h/L is very large and tanh kh tends toward unity. Under this condition (1) and (2) yield

$$C^2 = \frac{gL}{2\pi} = \left(\frac{gT}{2\pi}\right)^2 .$$

On the other hand, under the condition $h/L \leq 1/20$, (1) becomes to a good approximation

$$C^2 = gh .$$

There is, of course, no sharp distinction between "deep" and "shallow" water. The effect of depth of water on wave characteristics is gradual and waves in any finite depth of water are affected by that depth. It is interesting to note that waves classified in the shorter period category are not refracted until near shore and shallow water, while long period waves are refracted further offshore in deeper water. Tsunamis, for example, of periods 2 minutes to 1 hour would constantly undergo refraction effects in the open ocean. In a tsunami study, only waves that travel at a speed of $(gh)^{\frac{1}{2}}$ are considered.

As waves propagate into regions of changing depth, we consider that T remains unchanged while H and L vary.

Given an initially plane wave of a given period and wave height moving towards shallow water, the wave height in general can vary according to the following factors:

(a) Direct shoaling effects
(b) Wave refraction effects
(c) Reflection

(d) Diffraction
(e) Energy lost by friction and percolation

In this study, only the first two factors are considered over the sloping bathymetry, however, reflection is taken into account at the shore. The energy dispersion due to diffraction occurs in areas of appreciable convergence or divergence of energy and in shallow water zones such as behind breakwaters. The diffraction tends to spread the wave energy into potential shadow zones. Friction losses on slopes greater than 0.01 are considered negligible (Walsh, Reid, and Bader, 1962). Percolation is related to the bottom characteristics and is considered negligible with respect to friction (Reid and Bretschneider, 1953). In addition, external wind influences are neglected. Kajiura (1964) discusses the partial reflection of water waves passing over a bottom of variable depth. Green's formula provides a very good estimate of the transmitted wave amplitude even if partial reflection exists on a sloping bottom (Reid, 1957).

In the absence of energy losses by friction and percolation or dispersion by diffraction and reflection, the wave energy transmitted between orthogonals (wave rays) remains constant.

VERTICAL CROSS - SECTION

OF ISLAND

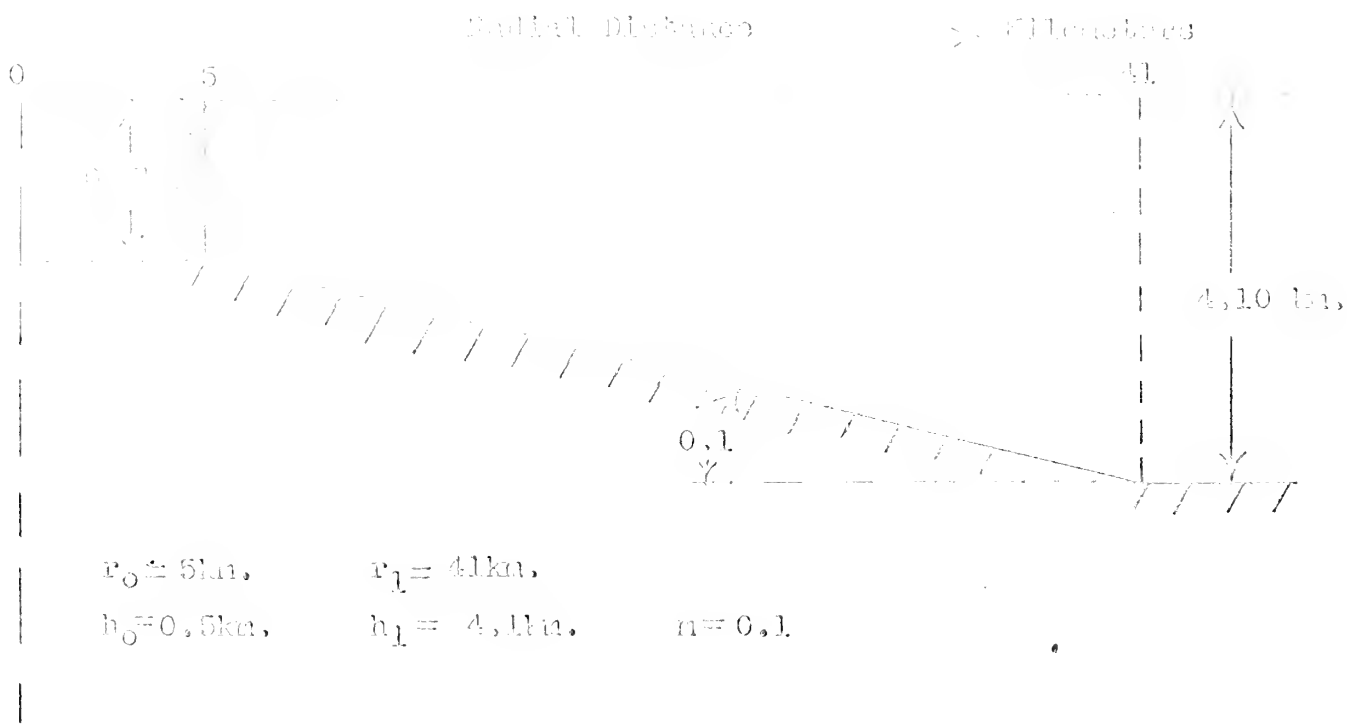


Figure 1.

CHAPTER II

THEORETICAL ASPECTS OF THE PROBLEM

1. Wave Refraction

Gravity wave refraction is analagous to the bending of light rays in geometrical optics. Since the wave speed decreases with decreasing depth ($C = (gh)^{\frac{1}{2}}$), the wave rays are refracted towards regions of lesser water depth.

This can be put in quantitative terms by the relation

$$\frac{d\beta}{ds} = -\frac{1}{C}\frac{\partial C}{\partial n} \tag{1.1}$$

where β is the angle of the wave ray measured counter-clockwise from a fixed reference (the x-axis), s is arc distance on the wave ray and n is distance measured parallel to the wave crest and to the left of the ray. The above relation can be shown to be consistent with Fermat's principle (Munk and Arthur, 1952). In the special case of straight and parallel depth contours, one can take the x-axis normal to the contours such that $C = C(x)$ alone. In this case (1.1) reduces to

$$\cos\beta \frac{d\beta}{dx} = \frac{1}{C}\sin\beta \frac{dC}{dx}$$

which is readily integrated and yields Snell's law-

$$\frac{\sin \beta}{C} = \text{constant} \tag{1.2}$$

along the wave ray. Note however, that this is a special case of the more general relation (1.1).

In the general case, graphical methods are often referred to in describing the refraction pattern. The two methods most frequently employed are: first, the crest method, where successive positions of the wave crest are drawn by plotting the wave advance from point to point along the crest; second, the ray method, where each orthogonal is plotted directly by determining its shoreward deflection as it crosses successive bottom contours. Johnson, O'Brien, and Isaacs (1948) provide explicit instructions for this approach. In both cases a detailed large scale chart of the bottom topography is essential. For the advantages and disadvantages of each method the reader is referred to an excellent discussion by Dunham (1950).

The refraction of energy in the lee of an island is difficult to determine by using a graphical approach. For this reason analytical solutions for the rays have been developed, where the depth variation is expressed as an analytic function (Arthur, 1951). Before proceeding with a discussion of this particular island case the relationship between wave height and energy will be

reviewed.

If it is assumed that the total energy transmitted between two orthogonals is constant, convergence of the rays denotes a concentration of energy and a corresponding increase in wave heights. To a good approximation, wave height is proportional to the square root of the energy provided that the waves are not near the breaking point (Munk and Traylor, 1947). The wave height, however, will not usually be the same along a particular wave crest due to refraction effects over an irregular bottom.

The mean wave energy per unit surface area for progressive waves is given by

$$E = 1/8\,\rho g H^2$$

where ρ is the density of sea water, g the acceleration due to gravity and H the wave height. The mean wave power, EVb is constant between two wave rays and it follows that the relationship for the wave height is given by

$$H = K_r K_s H_1 \tag{1.3}$$

where H_1 is the deep water wave height, K_r is the refraction factor and K_s is the shoaling factor.

The refraction factor is defined by

(1.4) $$K_r = (b_1/b)^{\frac{1}{2}}$$

where b is the distance between adjacent rays measured normal to the rays and the subscript 1 refers to deep water.

The shoaling factor can be expressed by

(1.5) $$K_s = (V_1/V)^{\frac{1}{2}}$$

where V_1 is the deep water group speed. In general

$$V = nC$$

where n is given by

$$n = \frac{1}{2}\left[1 + \frac{4\pi h/L}{\sinh \frac{4\pi h}{L}}\right] .$$

For the wave periods considered in this study n is essentially unity for all depths and hence V = C. Thus

(1.6) $$K_s = (h_1/h)^{\frac{1}{4}} .$$

For an incident wave of unit amplitude it follows from (1.3) and (1.6) that the relationship for relative amplitude is given by

(1.7) $$A = K_r(h_1/h)^{\frac{1}{4}} .$$

2. Fermat's Principle

Fermat's principle of least time is discussed in Joos (1934). It can be stated thus: the propagation of light always takes place in such a way that the actual optical path (e.g. length of geometric path multiplied by index of refraction of the medium) is an extreme value compared with all other paths which do not follow the law of optics. In a wave refraction analogy, we allow the light rays to correspond to orthogonals in a gravity wave refraction diagram.

The application of Fermat's least time concept to refraction by concentric, circular, bottom contours is discussed by Arthur (1946). The travel time along a path from a point A to a point B of a refracted ray is given by

$$I = \int_A^B ds/C \quad , \tag{2.1}$$

where $C = (gh)^{\frac{1}{2}}$ along the path and ds is the arc length along the path. Consider depth $h = h(r)$ only; then for I to be a minimum requires an $r(\theta)$ such that $\delta I = 0$. For $r > r_1$, the depth is assumed constant and the orthogonals are straight lines. For $r_0 < r < r_1$ the rays are curved. Using the coordinates as shown in Figure 3, (2.1) takes the form

$$(2.2) \qquad I = \int_{\theta_A}^{\theta_B} \frac{\sqrt{(dr/d\theta)^2 + r^2}}{C(r)}\, d\theta \quad .$$

A path which makes the integral of (2.2) a minimum is found by the methods of the calculus of variations (Joos, 1934), which in this case must satisfy the Euler-Lagrange condition

$$\frac{d}{d\theta}\left(\frac{1}{GC}\, dr/d\theta\right) - \frac{r}{GC} + G/C^2\, dC/dr = 0 \quad ,$$

where $G \equiv \left[r^2 + (r')^2 \right]^{\frac{1}{2}}$ and $r' = dr/d\theta$.

Let $F = G/C(r)$. For C independent of θ , F depends only on r and r' then

$$(2.3) \qquad \frac{d}{d\theta}\left[F - r' \frac{\partial F}{\partial r'} \right] = 0 \quad , \qquad r' \neq 0 \; ,$$

which implies that

$$F - r' \frac{\partial F}{\partial r'} = K, \text{ a constant.}$$

If the constant of integration is evaluated at $r = r_1$ the equation for the ray path takes the form

$$(2.4) \qquad \pm\, d\theta = dr/\left[r\sqrt{(r/r_1)^2 (1/\sin^2\theta_1)(h_1/h(r)) - 1} \right] ,$$

the sign being chosen according to the direction of the wave ray.

3. Ray Path

Consider the ray equation

$$(3.1) \qquad \frac{d\theta}{dr} = - \frac{1}{r\sqrt{\frac{\Gamma r^2}{h} - 1}} ,$$

where

$$\Gamma = \frac{h_1}{r_1^2 \sin^2 \theta_1} .$$

Now $h = mr$ for $r_o < r < r_1$, where m is the island slope. Thus

$$(3.2) \qquad \frac{\Gamma r^2}{h} = \frac{r}{r_1 \sin^2 \theta_1} .$$

Substituting (3.2) into (3.1) and carrying out the integration, with the boundary condition $\theta = \theta_1$ at $r = r_1$, gives

$$(3.3) \qquad \csc^2 \left(\frac{\theta_1 + \theta}{2} \right) = \frac{r}{r_1 \sin^2 \theta_1} .$$

The wave ray which enters at r_1, θ_1 will reach the island boundary, r_o, at $\theta = \theta_o$ where

$$(3.4) \qquad \theta_o = -\theta_1 + 2\operatorname{ctn}^{-1} \left(\frac{\sqrt{\frac{r_o}{r_1} - \sin^2 \theta_1}}{\sin \theta_1} \right) ,$$

provided that $\sin^2 \theta_1 \leq r_o/r_1$.

Consider $\sin^2 \theta_1 = r_o/r_1$, then

$$\theta_o = \pi - \theta_1 .$$

For $\sin^2 \theta_1 > r_o/r_1$, the ray does not reach the island perimeter. In this case, the ray becomes tangent to a circle of radius r_c given by

(3.5) $$r_c = r_1 \sin^2 \theta_1 ,$$

i.e. $r_c > r_o$ at $\theta = -\theta_1 + \pi$. A limiting ray exists at r_c maximum or when $\theta_1 = \pi/2$ (see Figure 4). Beyond this maximum critical radius, wave rays are unaffected by the island.

For $r_c = r_o$, the initial critical angle can be determined by

(3.6) $$\sin \theta_{1c} = \sqrt{\frac{r_o}{r_1}} .$$

For the particular island configuration considered here, $\theta_{1c} = 20.44^\circ$. Hence rays will converge on the island perimeter for $-159.56^\circ \leq \theta_o \leq 159.56^\circ$.

4. Refraction Factor

The refraction factor is given by the equation

(4.1) $$K_r^{-2} = \frac{r_o d\theta_o \cos\alpha_o}{r_1 d\theta_1 \cos\theta_1}$$

where $r_1 \cos\theta_1 d\theta_1$ is the mathematical expression for the ray separation at r_1 and $r_o d\theta_o \cos\alpha_o$ represents the wave separation at r_o. From (3.4)

(4.2) $$\frac{d\theta_o}{d\theta_1} = \frac{2\cos\theta_1}{\frac{r_o}{r_1} - \sin^2\theta_1} - 1 .$$

From Figure 5.

$$\tan\alpha = \frac{r d\theta}{dr} .$$

Substituting for $d\theta/dr$ from (3.1),

$$\tan\alpha_o = \frac{1}{\frac{\Gamma r_o^2}{h_o} - 1} .$$

By further trigonometric substitution for Γ

(4.3) $$\cos\alpha_o = \left(1 - \frac{r_1 \sin^2\theta_1}{r_o}\right)^{\frac{1}{2}} .$$

Substituting (4.2) and (4.3) into (4.1) gives

$$K_r^{-2} = \sqrt{\frac{r_o}{r_1}} \left[2 - \sec\theta_1 \sqrt{\frac{r_o}{r_1} - \sin^2\theta_1}\right] .$$

This is valid for $|\theta_1| \leq \theta_{1c}$.

5. Phase Lag

The water level displacement of a sinusoidal wave is given by

(5.1) $$\zeta = A \cos (\omega t - \phi)$$

where ϕ is the phase lag at any point on the ray relative to some fixed reference. The change in ϕ along a ray can be expressed mathematically by

(5.2) $$\frac{d\phi}{ds} = \frac{\omega}{C(r)}$$

where $ds = \left[(dr)^2 + (rd\theta)^2 \right]^{\frac{1}{2}}$.

The phase change along the path from $r = r_1$ to $r = r_o$ using (3.1) and (3.3) is given by

(5.3) $$\phi' = \frac{2\omega\sqrt{r_1}}{\sqrt{gh_1}} \left(\sqrt{r_1 - r_1 \sin^2 \theta_1} - \sqrt{r_o - r_1 \sin^2 \theta_1} \right).$$

A correction $\Delta\phi$ must be added for the change between the reference line $\phi = 0$ and $r = r_1$ (see Figure 6) where

(5.4) $$\Delta\phi = \frac{\omega r_1 (1 - \cos \theta_1)}{\sqrt{gh_1}}.$$

Thus for the total phase lag as a function of θ_1,

$$\phi = \frac{2\omega r_1}{\sqrt{gh_1}} \left[\tfrac{1}{2}(1+\cos\theta_1) - \sqrt{\frac{r_0}{r_1} - \sin^2\theta_1} \right] . \qquad (5.5)$$

6. Relative Power Distribution

The general relationship for the refraction factor for $r_o < r < r_1$ is given by

$$K_r^{-2} = \sqrt{\frac{r}{r_1}}\left[2 - \sec\theta_1\sqrt{\frac{r}{r_1} - \sin^2\theta_1}\right] .$$

Substituting for r from (3.2) gives

$$\text{(6.1)} \qquad K_r^{-2} = \sin\theta_1 \csc\left(\frac{\theta_1 + \theta}{2}\right)\left[2 - \tan\theta_1 \operatorname{ctn}\frac{\theta_1 + \theta}{2}\right],$$

valid for $\theta_1 < \theta < (2\pi - 3\theta_1)$. For $\theta = \theta_1$, $K_r^{-2} = 1$. For $\theta = \pi - \theta_1$, e.i. the critical angle for a minimum r of a ray, $K_r^{-2} = 2\sin\theta_1$. For $\theta = 2\pi - 3\theta_1$, $K_r^{-2} = 3$.

For direct rays characterized by a particular value of θ_1, K_r^{-2} is given by (6.1) for any θ along the ray path. For those rays with $|\theta_1| < \theta_{1c}$, reflection occurs at $r = r_o$, $\theta = \theta_o$. The reflected ray passes through $r = r_1$ at $\theta = 2\theta_o - \theta_1$ (see Figure 8). It emerges straight beyond r_1 and makes an angle $2\theta_o$ from the x-axis. Thus two rays θ_1 and θ_1' diverge at angular spread $2(\theta_o' - \theta_o)$ and their intersection always lies within a circle of radius r_1. Thus for $r >> r_1$ at $|\theta| = 2\theta_o$ and $|\theta_1| < \theta_{1c}$

$$K_r^2 = \frac{r_1 \cos \theta_1 d\theta_1}{2rd\theta_o} \quad .$$

Substituting from (4.1)

$$(6.2) \qquad K_r^2 = \frac{r_1 \cos \theta_1}{2r\left(\dfrac{2\cos \theta_1}{\sqrt{\dfrac{r_o}{r_1} - \sin^2 \theta_1}} - 1\right)}$$

For those rays with $\pi/2 > |\theta_1| > \theta_{1c}$ no reflection occurs. The rays are refracted and emerge at $r = r_1$, $\theta = 2\pi - 3\theta_1$ as a straight line and make an angle of $2\pi - 2\theta_1$, from the x-axis. Thus two neighboring rays θ_1 and θ_1' have a divergent angle $2(\theta_1' - \theta_1)$ and intersect within a circle of radius r_1. For $r >> r_1$ at $|\theta| = 2\pi - 2\theta_1$, $\theta_{1c} < |\theta| < \pi/2$. Then

$$(6.3) \qquad K_r^2 = \frac{r_1 \cos \theta_1}{2r} \quad .$$

For any given radial distance, r, from the center of the island, the relative power intensity can be expressed in the form

$$(6.4) \qquad F(\theta) = \frac{r}{r_1} K_r^2 \quad .$$

The form of this "beam pattern" for reflected and refracted waves in the far field is determined by (6.2) and (6.3), respectively. The total, relative power intensity in the far field is the sum of that contributed by the

reflected and refracted waves along radial lines of θ values (see Figures 9 and 10 in Appendix II).

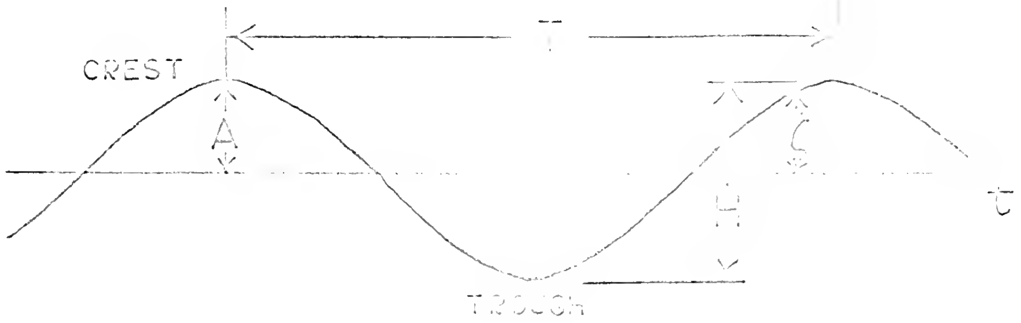


FIGURE 2--WAVE DEFINITIONS

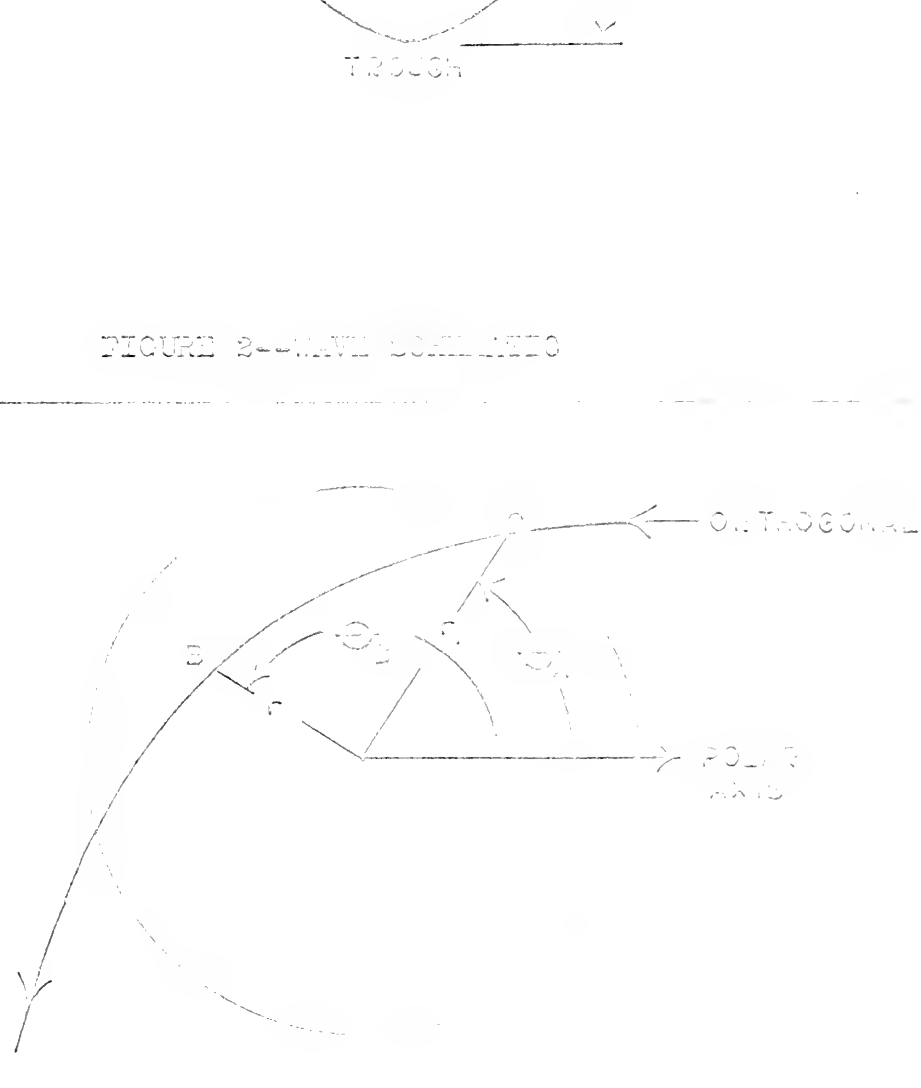


FIGURE 3--POLAR COORDINATES

SCHEMATIC DIAGRAM - CIRCULAR ISLAND

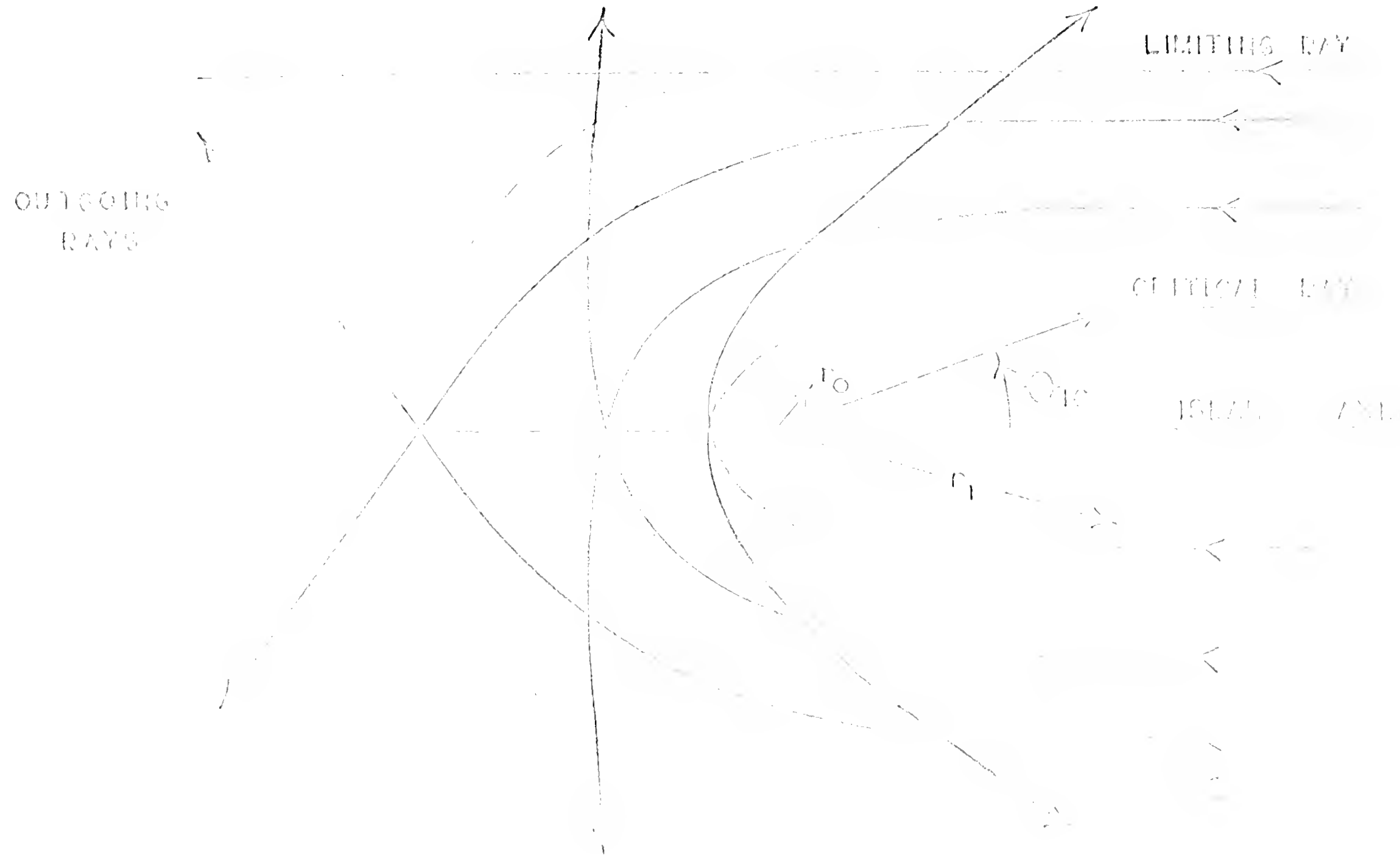
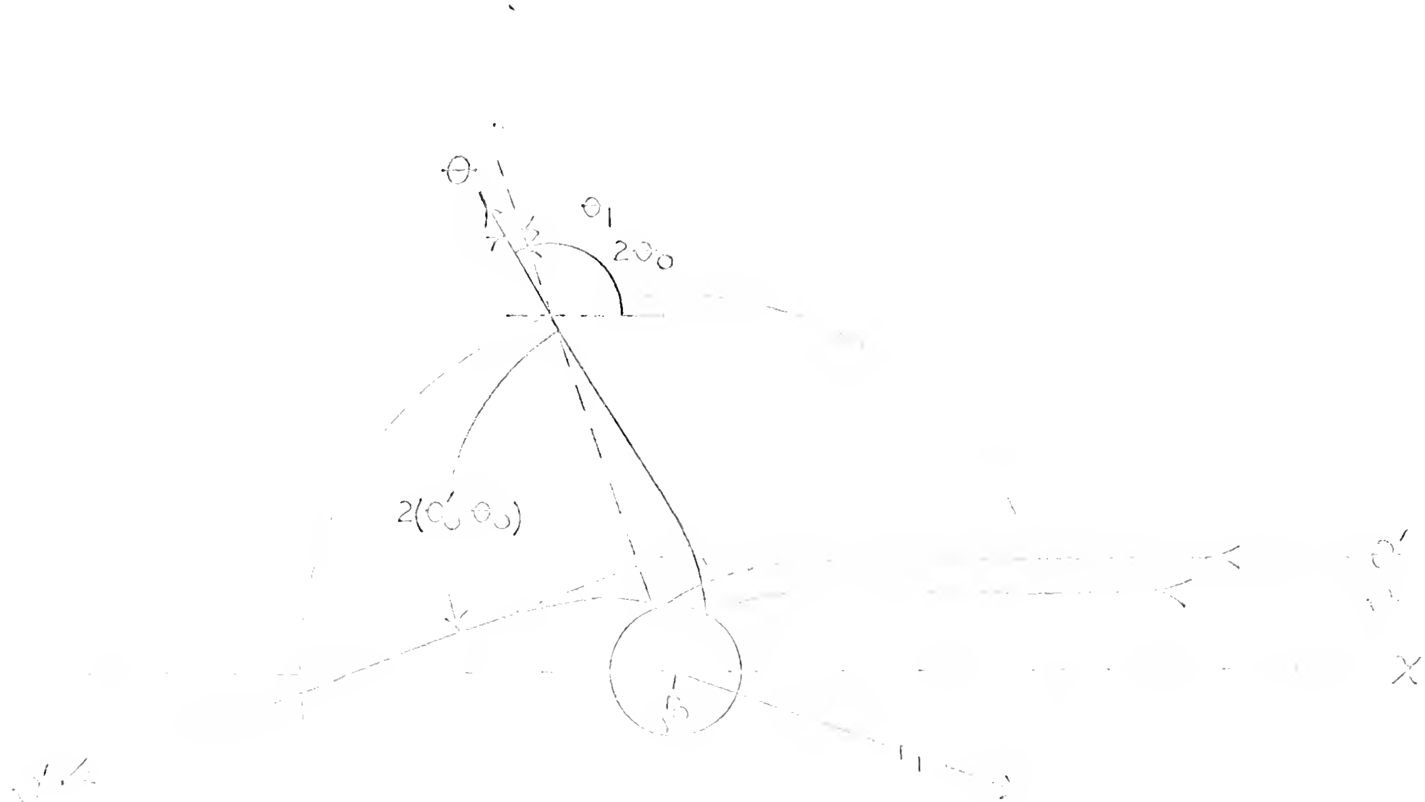


FIGURE 4.



C H A P T E R III

DISCUSSION OF RESULTS

A calculation summary for the circular island appears in Appendix I. This is presented in two parts: (A) information pertinent to the rays and refraction on the island lee shore; (3) information pertinent to the far field power distribution. The graphical representation of phase lag, refraction factor, relative amplitude, and relative power intensity for the island shore are given in Figures 9 to 12 (Appendix II). A simple wave refraction pattern consisting of direct rays into the island shore is produced by the constant slope case. For those incident waves entering with azimuths within the range from -20.4 to 20.4 degrees, the waves converge on the island covering an angular range from 0 to $\pm$ 15[illegible] degrees. A wave shadow zone of about 40 degrees range is formed in the lee of the island and is an area unaffected by refraction effects. The shadow zone is developed between the shore and the outgoing critical ray.

For those rays with $|\theta_1| < \theta_{1c}$, reflection occurs at $r = r_o$, $\theta = \theta_o$ and the ray emerges from r_1 at $\theta = 2\theta_o - \theta_1$, making an angle of $2\theta_o$ with the x-axis. For those rays with $\pi/2 > |\theta_1| > \theta_{1c}$, no reflection occurs, but the rays are refracted and finally emerge

at $r = r_1$, $\theta = 2\pi - 3\theta_1$ and with an angle of $2\pi - 2\theta_1$ from the x-axis. At the point of emergence $K_r = 1/\sqrt{3}$, however, at $\theta_1 = \pm \pi/2$, a discontinuity exists. Incident rays that are just outside r_1 at $\pm \pi/2$ are unaffected by the island, while those rays just inside r_1 undergo refraction.

For those emergent rays at large distance from the island ($r >> r_1$), the center of the island can be considered as the origin of these rays and the relative power intensity in the far field (rK_r^2/r_1) is simply a function of θ.

The relative power intensity is found to increase gradually from $|\theta| = 180$ to an extreme at $2\theta_{1c}$ (40.8 degrees) at which there is an abrupt decrease. This discontinuity in the far field beam pattern of the power is caused by the shadow phenomenon inherent in this refraction analysis. The contribution of the refracted waves in the far field exceeds that of the reflected rays by more than a factor of 4 (see Figures 13 and 14).

On the other hand, when considering the wave amplitude at the island shore, derived by use of Green's Law, the shoaling factor is significant while refraction effects are limited by the small θ_{1c} value. Furthermore, for a constant slope of 0.1, any variation in the

island contour parameters will not appreciably alter the critical angle. The refraction factor varied a small amount due to the slope and parameters of the island selected (see Figure 10).

The variation in relative amplitude, in this case, is governed by the refraction factor and drops off abruptly to zero at the point of ray tangency to the island. The energy for the near critical rays does not dissipate at a single point on the island shore as suggested by the refraction analysis. It is in this region of energy convergence that the assumption of constant power between wave rays is no longer valid and a down gradient flow of energy occurs across orthogonals. The "shadow zone" of the island is no longer a region of constant energy flux, but would contain energy input lost by diffraction effects near the point of tangency of the critical ray. Thus, by conducting a refraction analysis alone, it is difficult to adequately describe the energy distribution on the wave lee side of the island. This also applies to the effective shadow zone in the far field pattern.

On the far side of the island, where a diverging cross-over pattern of emerging rays is formed, the relative amplitude can be derived by combining that of two outgoing wave rays at their proper phase relationship.

The phase lag on the island shore increases with θ_o (see Figure 11). The range of phase lag at the island shore varies from about 1 radian for waves of 12 minute period to about 6.5 radians for waves of 2 minute period. Thus the phase lag of waves in the lee of the island varies considerably with period, while the amplitude is unaffected by the period (within the tsunami range).

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APPENDIX I

CALCULATION SUMMARY

A. CALCULATION SUMMARY FOR THE ISLAND SHORE

$r_o = 5$ km. $r_1 = 41$ km. $h_o = .5$ km. $h_1 = 4.1$ km.

θ_o deg.	θ_1 deg.	K_r	ϕ T=2 min (rad)	ϕ T=4 min (rad)	ϕ T=8 min (rad)	ϕ T=12 min (rad)	A
00	00.00	1.3170	14.40	7.20	3.60	2.40	2.2284
10	2.10	1.3155	14.45	7.21	3.61	2.41	2.2258
20	4.2	1.3136	14.51	7.29	3.65	2.42	2.2226
30	6.2	1.3110	14.69	7.35	3.70	2.45	2.2182
40	8.18	1.3061	14.89	7.47	3.76	2.49	2.2099
50	10.08	1.3000	15.12	7.56	3.81	2.51	2.1996
60	11.8	1.2935	15.46	7.70	3.89	2.59	2.1886
70	13.41	1.2856	15.85	7.89	3.96	2.62	2.1752
80	14.90	1.2768	16.25	8.07	4.06	2.71	2.1603
90	16.18	1.2670	16.70	8.30	4.15	2.80	2.1438
100	17.30	1.2575	17.19	8.59	4.30	2.89	2.1277
110	18.30	1.2475	17.70	8.88	4.43	2.98	2.1108
120	19.06	1.2365	18.30	9.17	4.60	3.08	2.0922
130	19.68	1.2266	18.89	9.48	4.75	3.18	2.0754
140	20.01	1.2165	19.50	9.80	4.90	3.26	2.0583
150	20.32	1.2065	20.15	10.11	5.05	3.38	2.0414
159.56	20.44	1.1970	20.87	10.41	5.20	3.47	2.0253
170	SHADOW ZONE						
180							

B. CALCULATION SUMMARY FOR THE FAR FIELD OF THE ISLAND

θ deg.	θ_o deg.	θ_1 deg.	$\left(\frac{r}{r_1}Kr^2\right)$ total refl.	$\left(\frac{r}{r_1}Kr^2\right)$ refr.	$\left(\frac{r}{r_1}Kr^2\right)$ total
0	0	0	.1057		.1057
20	10	2.1	.1054	shadow	.1054
40	20	4.2	.1032		.1032
40.8	20.4	4.21	.1032	.468	.5712
60	30	6.20	.1089	.433	.5419
80	40	8.18	.1128	.383	.4958
100	50	10.08	.1136	.321	.4346
120	60	11.80	.1162	.250	.3662
140	70	13.41	.1168	.171	.2878
160	80	14.90	.1177	.087	.2047
180	90	16.18	.1186	0	.1186
200	100	17.30	.1177	.087	.2047
220	110	18.30	.1168	.171	.2878
240	120	19.06	.1162	.250	.3662
260	130	19.68	.1136	.321	.4346
280	140	20.01	.1128	.383	.4958
300	150	20.32	.1089	.433	.5419
319.2	159.6	20.44	.1032	.468	.5712
320	160		.1032	shadow	.1032
340	170		.1054		.1054

APPENDIX III

GRAPHICAL REPRESENTATION OF RESULTS

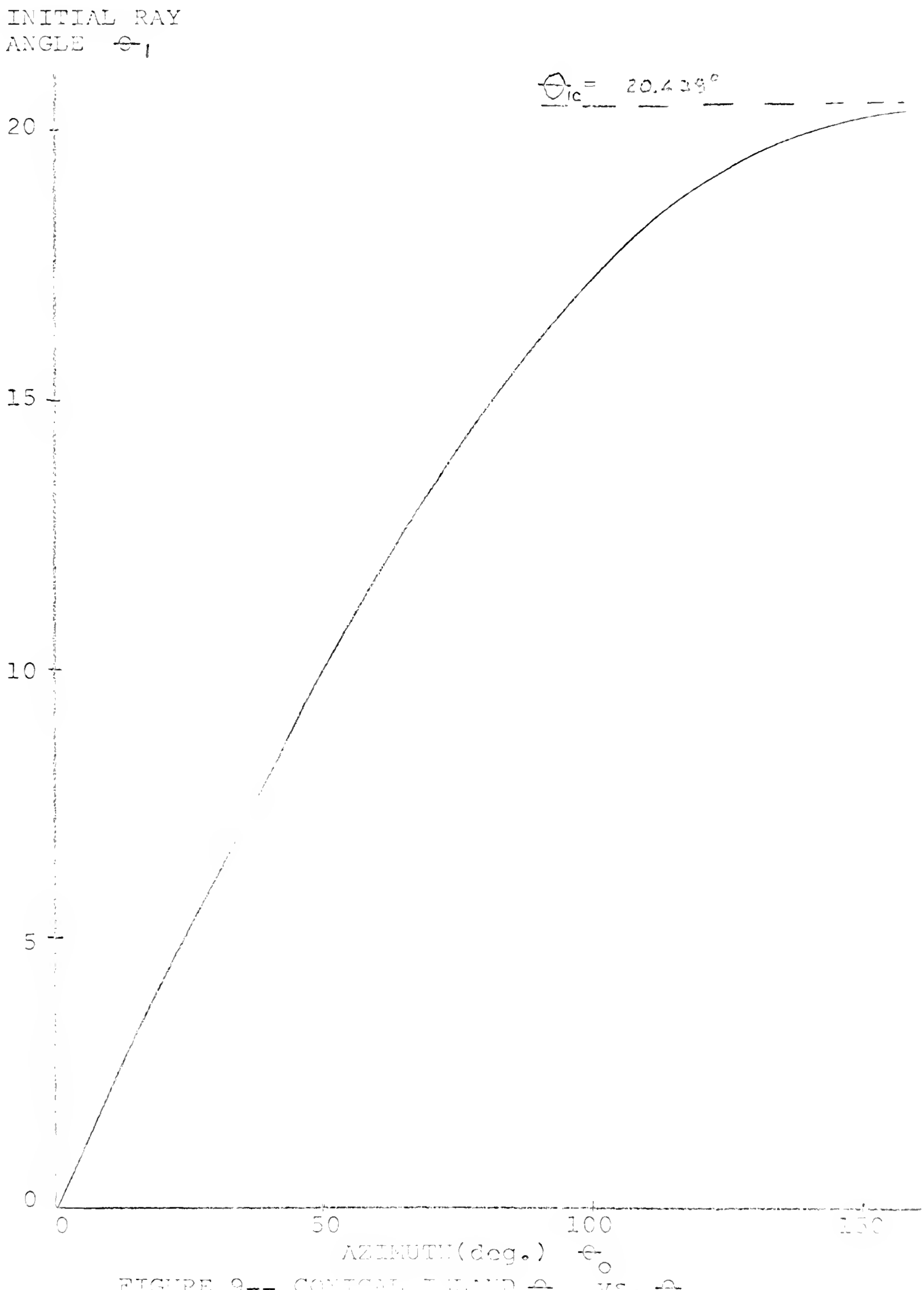

FIGURE 9-- CONICAL ISLAND θ_1 vs. θ_o

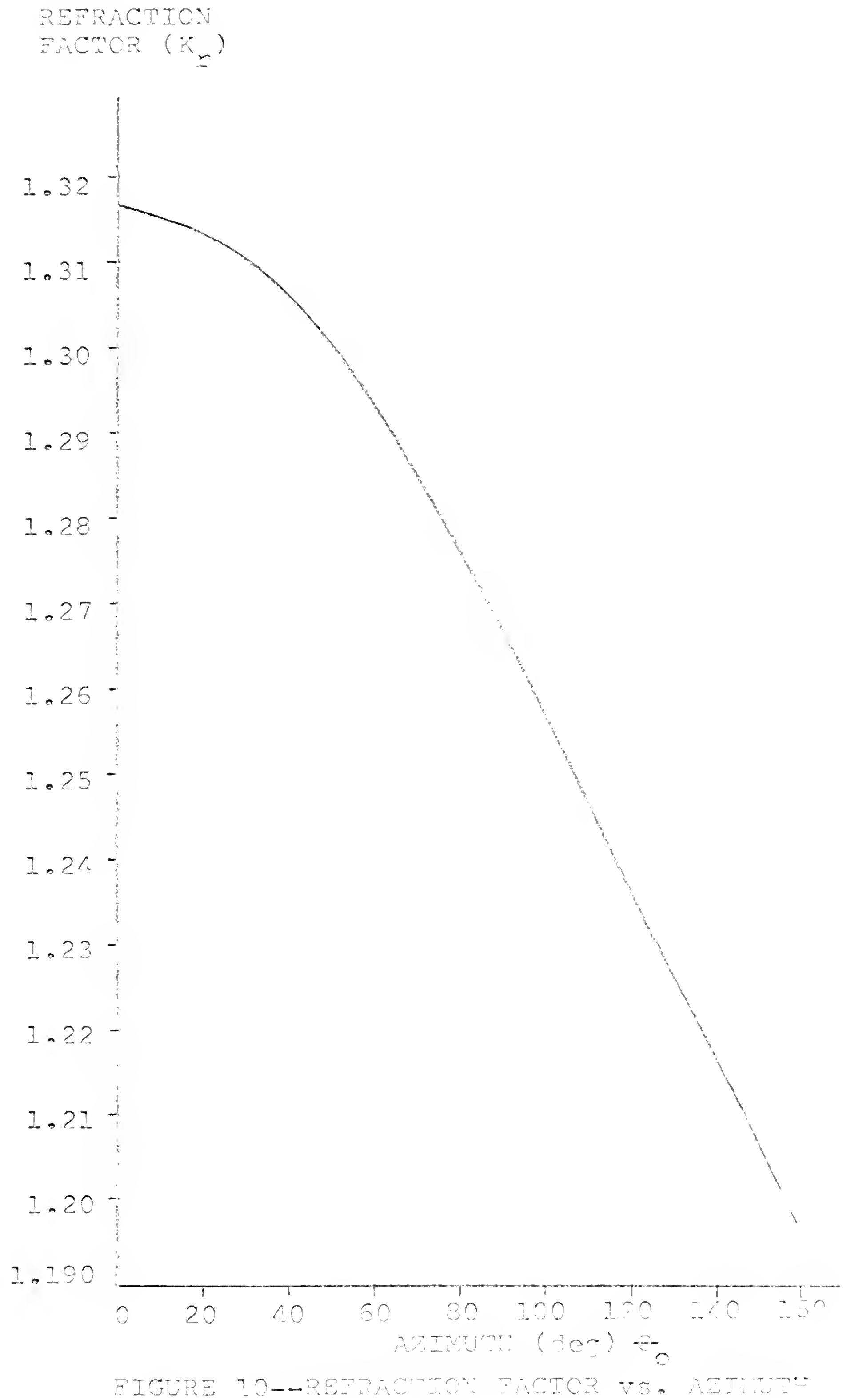

FIGURE 10--REFRACTION FACTOR vs. AZIMUTH

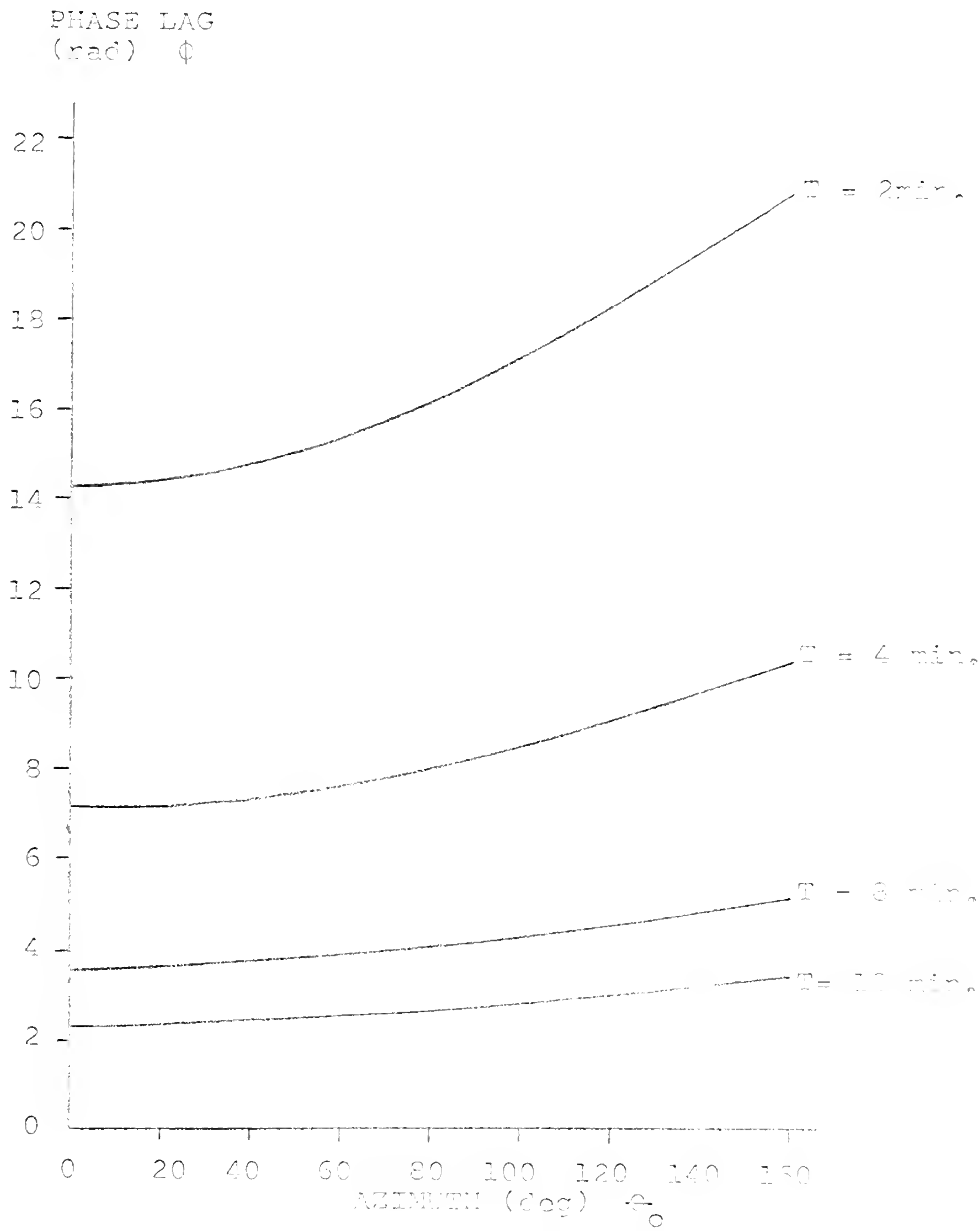

FIGURE 11--PHASE LAG vs. AZIMUTH

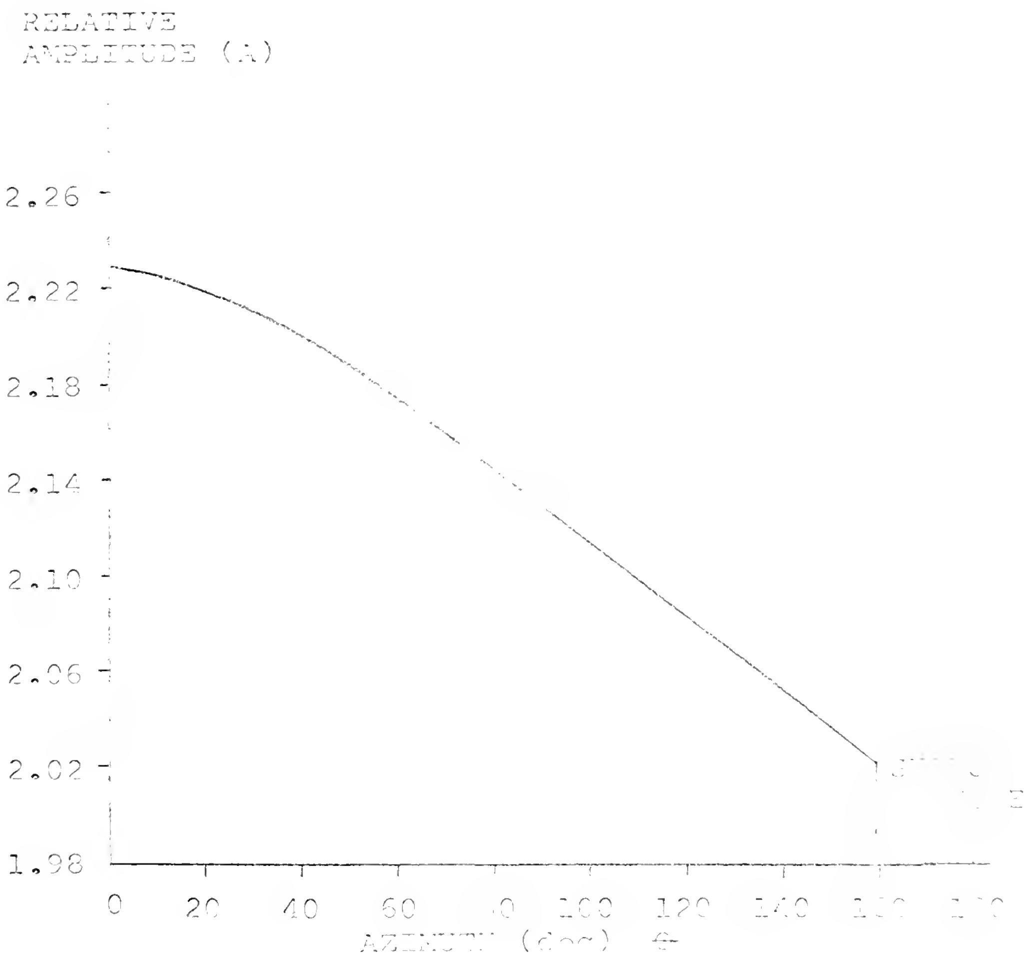

FIGURE 12--RELATIVE AMPLITUDE vs. AZIMUTH

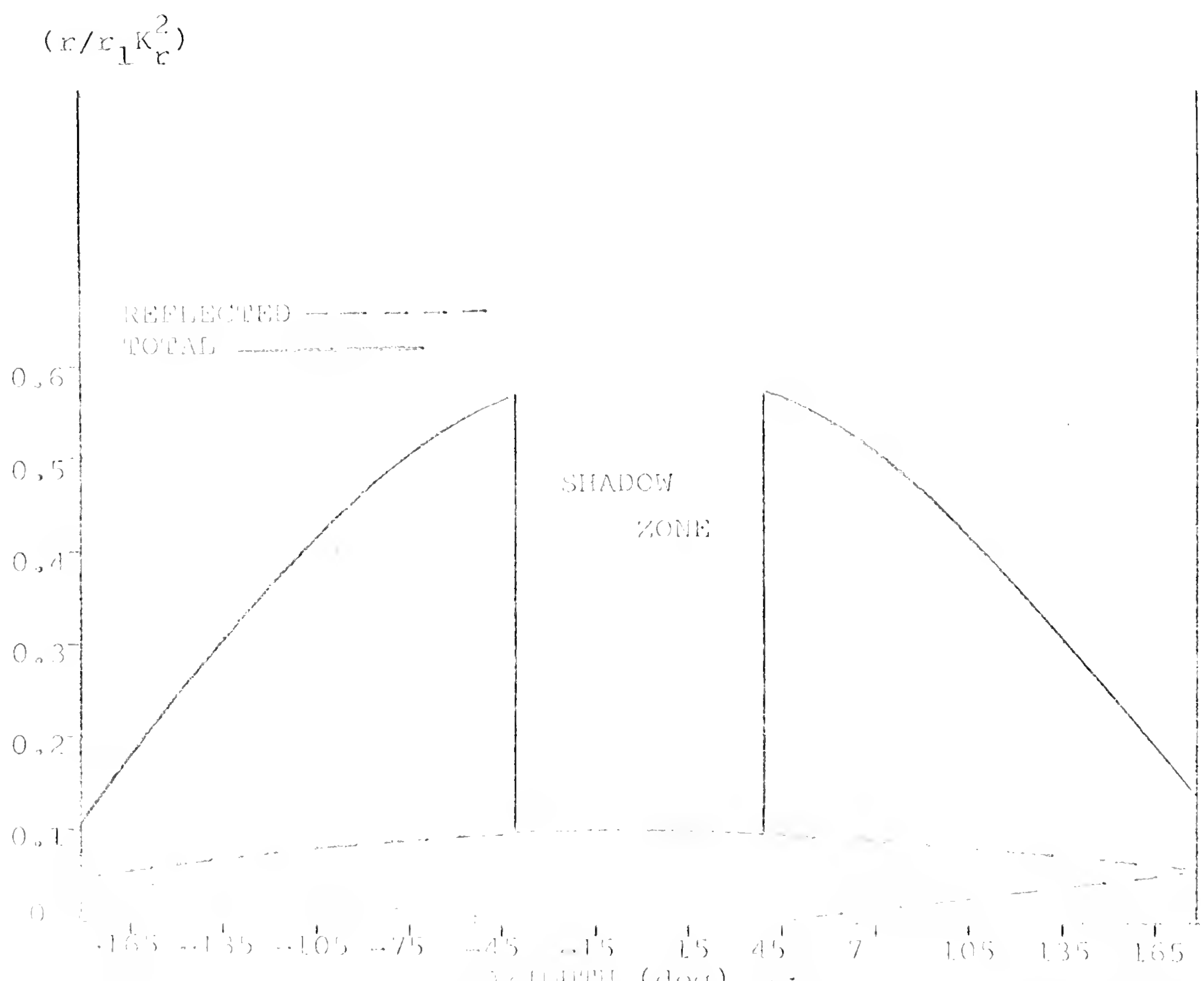

FIGURE 14.–TOTAL AND REFLECTED POWER INTENSITY vs. EMERGENT ANGLE

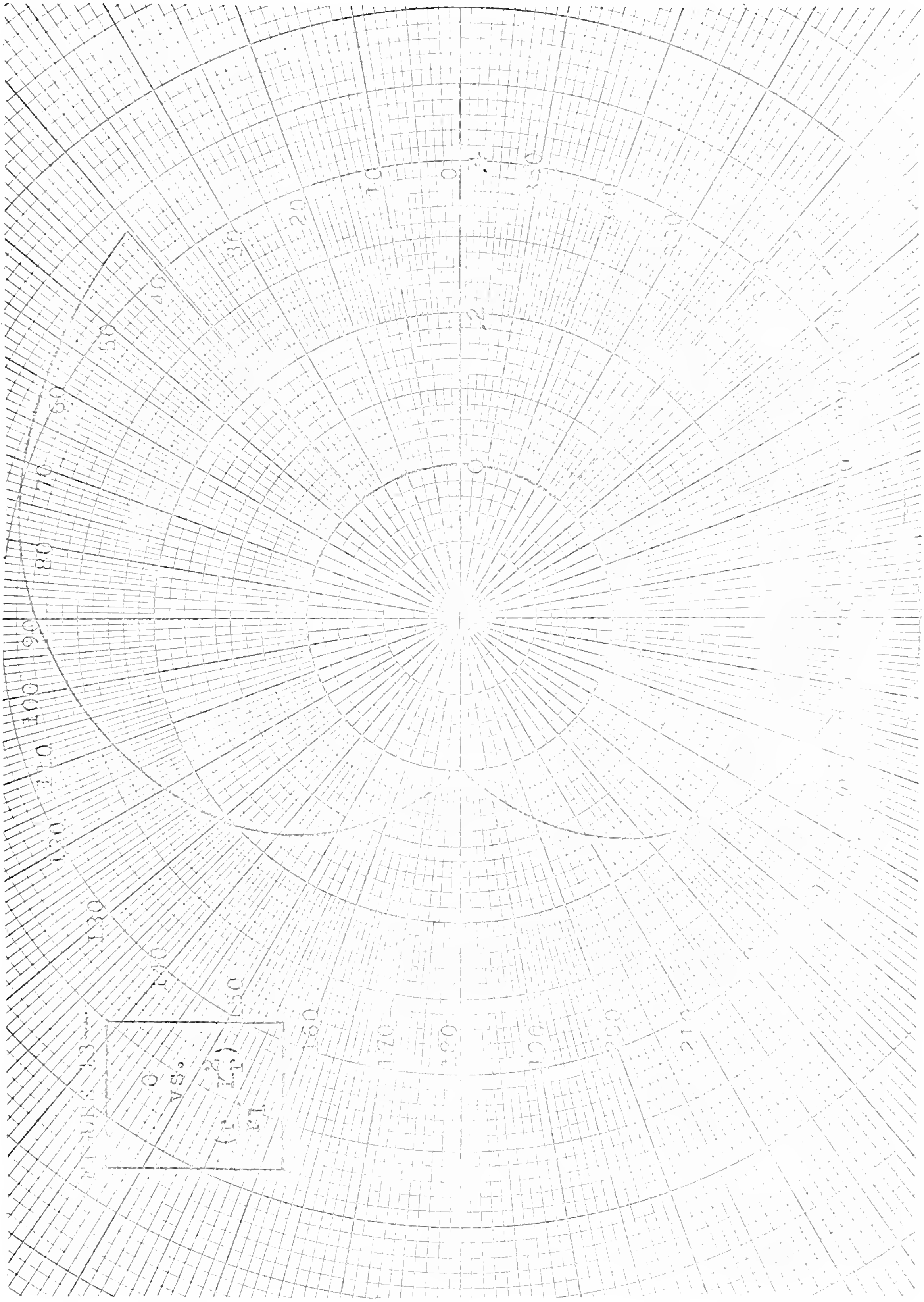

FIGURE 13